University of Victoria | Graduate Studies

Notice of the Final Oral Examination
for the Degree of Master of Science

of

ANTHONY JOHN CECIL

MA (University of Oxford, 1997)

"Lie Isomorphisms of Triangular and Block-Triangular Matrix Algebras over Commutative Rings"

Department of Mathematics and Statistics

Friday, August 5th, 2016
2:30pm
David Strong Building
Room C114

Supervisory Committee:
Dr. Ahmed R. Sourour, Department of Mathematics and Statistics, University of Victoria (Supervisor)
Dr. John Phillips, Department of Mathematics and Statistics, UVic (Member)

External Examiner:
Dr. Venkatesh Srinivasan, Department of Computer Science, UVic

Chair of Oral Examination:
Dr. Daniela Damian, Department of Computer Science, UVic

Dr. David Capson, Dean, Faculty of Graduate Studies

Abstract

For many matrix algebras, every associative automorphism is inner. We discuss results by Đoković that a non-associative Lie automorphism φ of a triangular matrix algebra T_n over a connected unital commutative ring, is of the form $\varphi(A) = SAS^{-1} + \tau(A)I$ or $\varphi(A) = -SJA^{\top}JS^{-1} + \tau(A)I$, where $S \in T_n$ is invertible, J is an antidiagonal permutation matrix, and τ is a generalized trace. We incorporate additional arguments by Cao that extended Đoković's result to unital commutative rings containing nontrivial idempotents.

Following this we develop new results for Lie isomorphisms of block upper-triangular matrix algebras over unique factorization domains. We build on an approach used by Marcoux and Sourour to characterize Lie isomorphisms of nest algebras over separable Hilbert spaces.

We find that these Lie isomorphisms generally follow the form $\varphi = \sigma + \tau$ where σ is either an associative isomorphism or the negative of an associative anti-isomorphism, and τ is an additive mapping into the center, which maps commutators to zero. This echoes established results by Martindale for simple and prime rings.